Analysis of Surface Mounted Permanent Magnet Motors Using Combined Winding Function and Conformal Mapping Method

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The conventional winding function theory (WFT) has main defects in the modeling of air-gap and magnetic saturation effect. Therefore, this paper presents an improved winding function theory (IWFT) for performance analysis of surface mounted permanent magnet (SMPM) motors, which removes all drawbacks of the conventional WFT. IWFT can precisely take into account the stator slotting, the winding distribution, the magnetic induction inside permanent magnets (PMs), the PM magnetization, and the magnetic saturation, simultaneously. To consider the harmonics of air-gap permeance, IWFT gets help from the conformal mapping (CM) method. The accuracy of this developed method is verified by comparing the results obtained through IWFT with the corresponding results obtained by conventional WFT, and finite element method (FEM).

***Index Terms*— Conformal mapping, magnetic saturation, permanent magnet , winding function theory.**

I. Introduction

THE ACCURATE and fast modeling techniques are necessary for design, optimization, and performance analysis of electrical machines. Finite elements method (FEM) is is more accurate than the analytical and semi-analytical methods; however, it is a time-consuming method [1]. The FEM packages also prevent engineers from having the profound knowledge about the performance of electrical machines. At this end, the analytical and semi-analytical techniques such as winding function theory (WFT) [2], magnetic equivalent circuit [3], conformal mapping (CM) [4], subdomain model [5], and field reconstruction method [6] have been introduced. However, the analytical and semi-analytical methods usually have imperfections in modeling of air-gap, magnetic saturation, and permanent magnets (PMs). To enhance the accuracy of analytical methods and to extend their applications, they may be combined to form hybrid analytical models (HAM) [7]. In this paper, a new HAM based on WFT and CM is presented, which is called improved winding function theory (IWFT).

II. IMPROVED WINDING FUNCTION THEORY

WFT is used to calculate the phase inductances as follows [2]:

$$L_{x,y} = \mu_0 . r . l_z \int_0^{2\pi} \frac{N_x(\varphi).n_y(\varphi)}{g(\varphi)} d\varphi \quad (1)$$

where r is the average radius in the air-gap, l_z is the axial length of the core, $N_x(\varphi)$ is the winding function of phase x, $n_y(\varphi)$ is the turn function of phase y, $g(\varphi)$ is the distribution of air-gap length, and $L_{x,y}$ is the mutual inductance of phases x and y. Eqn. (1) shows that the accuracy of the calculated inductance depends on the modeling accuracy of $g(\varphi)$, $N_x(\varphi)$, and $n_y(\varphi)$. In the previous publications, the non-uniformity in air-gap due to the slotting effect, salient poles, and the magnetic saturation were modeled with low accuracy using WFT [2]. This paper improves the accuracy of WFT by exact modeling of the slotting effect, PMs, and magnetic saturation in calculation of inductance.

A. Modeling Permanent Magnet

In conventional WFT, PM magnetization has been modeled with a low accuracy. For example, the parallel magnetized PMs were modeled with trapezoidal magneto-motive force functions [8].

In IWFT, each PM is replaced by the virtual one-turn coils with the constant current for each coil. The virtual coils are determined while having the PM equivalent surface currents. Fig. 1 shows the equivalent virtual coils for typical PMs with radial and parallel magnetizations (coils a_1a_1', …, a_4a_4', b_1b_1', …, b_4b_4', c_1c_1', …, c_4c_4'). The current of the virtual coils are calculated as follows:

$$I = \left|\vec{M} \times \vec{a_n}\right|. dl \quad (2)$$

where $\vec{M}$ is the magnetization vector, $\vec{a_n}$ is the unit normal vector to the surface of PM, and dl is the length of equivalent PM element. The PM magnetization level $(\vec{M})$ is updated in each rotor position while having the magnetic induction inside PM due to

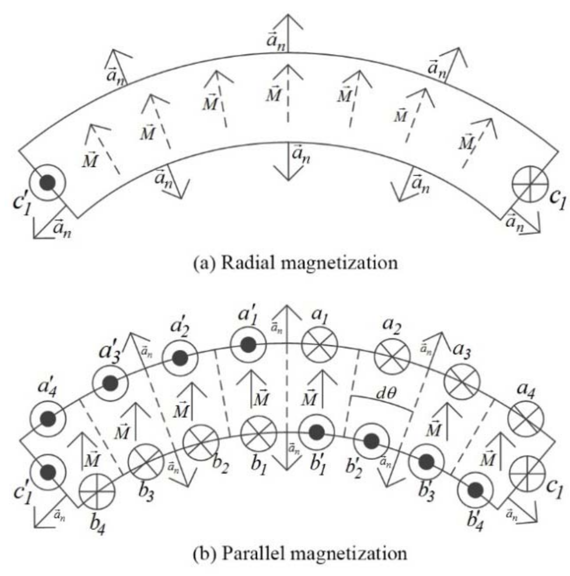

Fig. 1. Equivalent virtual coils for PMs.

the armature reaction. Therefore, the PM excitations are replaced by the coil excitations. The self and mutual inductances of rotor and stator coils are then calculated using IWFT for each rotor position.

B. Modeling Slotting Effect

Generally in the application of WFT, the slotting effect was either neglected [2] or considered with low accuracy by using the tooth contour method [9]. In [8], a single magneto-static FEM was used to calculate $g(\varphi)$, which makes WFT dependent on FEM. To consider the slotting effect, the radial component of the specific complex air gap permeance is calculated using the CM method $(\lambda_r(\varphi))$. The distribution of slotted air-gap length $g(\varphi)$ is then calculated as follows:

$$g(\varphi)=l_g \times (\lambda_r(\varphi))^{-1} \quad (3)$$

where l_g is the air gap length neglecting the stator slots.

Fig. 2 shows $g(\varphi)$ for one typical surface-mounted PM (SMPM) motor with 12 slots and 4 poles ignoring the magnetic saturation in one pole pitch. In Fig. 2, R is the radius of contour in the air gap, R_r is the outer radius of the rotor core, R_m is the outer radius of the rotor PMs, and R_s is the inner radius of the stator core.

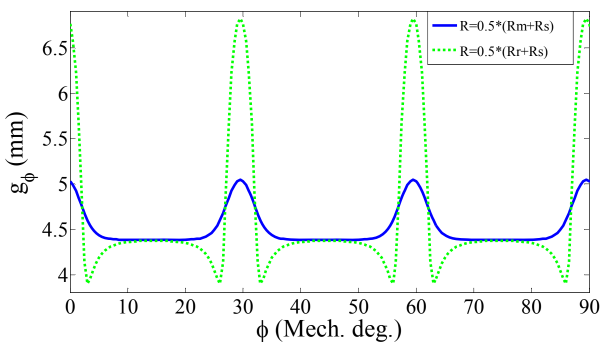

Fig. 2. Distribution of air-gap length.

C. Modeling Saturation Effect

The basic assumption of WFT is an infinite relative permeability of the core. Here, a new technique is presented that retains the basic assumption of WFT and considers the magnetic saturation by proper increase in the air gap length in the front of the stator teeth. To this end, a virtual one-turn coil is wound on each tooth. The current for each virtual tooth-coil is considered equal to the MMF of the respective tooth. The inductance matrix and the flux-linkage are then calculated for the virtual tooth-coils. The proper increase in the air gap length (Δl_g) is then determined for each stator tooth while having the flux-linkage of respective tooth-coil, as follows:

$$B_t = \frac{\lambda_t}{A_t} \xrightarrow{B-H\ cureve} \mu_{r,t} = \frac{1}{\mu_0}\frac{dB}{dH} \rightarrow$$
$$\rightarrow R_t = \frac{l_t}{\mu_0 \mu_{r,t} A_t} = \frac{\Delta l_g}{\mu_0 A_t} \rightarrow \Delta l_g = \frac{l_t}{\mu_{r,t}} \quad (4)$$

where index t is the abbreviation of tooth. l_t, A_t, and $\mu_{r,t}$ are the length, area and the equivalent μ_r of the stator tooth, respectively. Fig. 3 compares the PM flux-linkage of phase A obtained by WFT, IWFT, and FEM while considering the radial magnetization for PMs. Table I shows the some

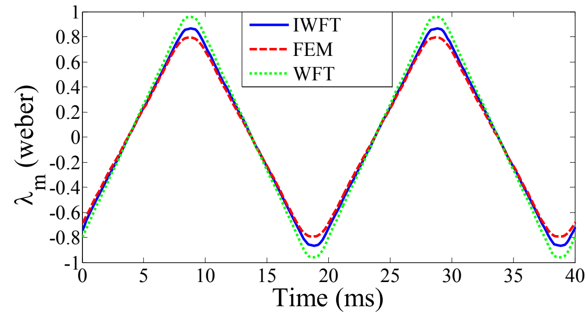

Fig. 3. PM flux-linkage of phase A.

TABLE I
MAIN PARAMETERS OF SMPM MOTOR

Parameter	Value	Parameter	Value (mm)
Number of pole pairs, p	2	Stator outer diameter	130
Number of slots, Q_s	12	Stator inner diameter	75
Magnet remanence, B_r	0.96 T	Active length, l_z	65
Relative recoil permeability, μ_r	1.07	Air gap length, g	1
Pole arc coefficient, αp	0.9	Magnet thickness	3.5

parameters of the analyzed SMPM motor.

III. CONCLUSION

This paper presents an improved WFT (IWFT) which is more accurate than the conventional WFT in modeling the slotting effect, magnetic saturation, and PMs. IWFT can accurately model the electric machines in short time, without the help of FEM, and by using very compact programs of Matlab.

IV. ACKNOWLEDGEMENT

The authors would like to thank Iran's National Elites Foundation (INEF) for financial support of the project.

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